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The Effect of Suction on Mixed Convection of a Hybrid Nanofluid Flow along a Vertical Surface Embedded in a Porous Medium

Nurul Athirah Md Shukri[1], Haliza Rosali[2*] and Nadihah Wahi[3]
[1,2,3]*Department of Mathematics and Statistics, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor*

[1]212320@student.upm.edu.my, [2]liza_r@upm.edu.my, [3]nadihah@upm.edu.my
*Corresponding author

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ABSTRACT
This study investigates the effect of suction on mixed convection of a hybrid nanofluid flow along a vertical surface embedded in a porous medium. Hybrid nanofluids, consisting of alumina and copper nanoparticles dispersed in a base fluid, are recognized for their superior thermal properties. The research emphasizes on the combined influence of suction and nanoparticle interactions on boundary layer behavior and heat transfer efficiency. By applying similarity transformations, the governing equations are reduced to ordinary differential equations and solved numerically using a boundary value problem solver. The results show that suction significantly enhances heat transfer rates by thinning the thermal boundary layer and increasing the temperature gradient near the surface. Variations in suction intensity and nanoparticle volume fractions directly affect on the skin friction coefficient and local Nusselt number, with copper nanoparticles contributing to more efficient heat dissipation due to their higher thermal conductivity. The existence of dual solutions reveals both stable and unstable flow regimes, underscoring the importance of parameter optimization for engineering applications. These findings offer insights into the design of advanced thermal management systems, demonstrating the potential of hybrid nanofluids and suction techniques in improving heat transfer performance in porous media.

Keywords: Hybrid Nanofluid, Mixed Convection, Suction, Boundary Layer, Porous Medium

INTRODUCTION

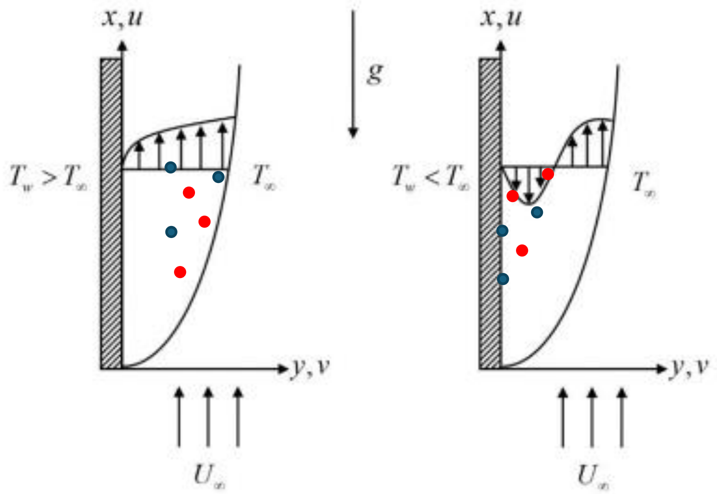
Improving heat transfer in fluid flow systems is a crucial area of research due to its wide-ranging applications in engineering, industry, and environmental processes. In particular, the study of boundary layer flow in porous media is essential for applications such as thermal insulation, geothermal energy extraction, and biomedical engineering. With the advancement of nanotechnology, nanofluids, which are engineered by dispersing nanoparticles in conventional fluids, have been introduced to enhance thermal performance. More recently, hybrid nanofluids, consisting of two or more different types of nanoparticles, have been developed to provide superior thermal conductivity, stability, and efficiency compared to single particle nanofluids (Aly et al., 2003; Abdulhadi et al., 2021; Jeelani & Abbas, 2023). Ahmad et al. (2012) studied the steady mixed convection boundary layer flow from a vertical cone in a porous medium filled with a nanofluid using different types of nanoparticles. Mahian et al. (2013) investigated the effects of nanofluids on the efficiency improvement of solar collectors as well as on economic and environmental considerations regarding the usage of a system. The steady mixed convection flow along a vertical surface embedded in a porous medium with hybrid nanoparticles was instigated by Waini et al. (2020). They found that the added hybrid nanoparticles delay the separation of the boundary layer. Hybrid nanofluids are highly regarded for their superior thermal properties, making them ideal for use in heat transfer systems such as cooling technologies and thermal insulation. These fluids, which consist of two or more types of nanoparticles suspended in a base fluid, have been shown to offer enhanced heat transfer performance compared to conventional fluids. Despite these advantages, the influence of suction on the boundary layer behavior of hybrid nanofluids, particularly in porous media, remains inadequately studied.

Among the mechanisms that influence heat transfer in nanofluid systems, suction is recognized as a vital factor. The suction parameter plays a crucial role in modifying fluid flow patterns and heat transfer rates, especially in systems where both forced and natural convection coexist. Suction refers to the removal of fluid through a porous surface, which helps control boundary layer formation, reduce thermal resistance, and improve heat dissipation. Its application enhances flow stability and temperature distribution in mixed convection flows along vertical surfaces embedded in porous media. Despite its importance, relatively limited attention has been given to investigating suction effects in hybrid nanofluid systems, particularly in porous environments. Previous studies have addressed nanofluid and hybrid nanofluid flows with suction or injection in different settings (Yasin, 2013; Bakar et al., 2022; Rubaa'i et al., 2022; Jeelani & Abbas, 2023). A stagnation point flow and heat transfer of a hybrid nanofluid over an exponentially shrinking sheet with suction/injection was investigated by Anuar et al. (2019) using similarity transformations and numerical methods. They found that the dual solutions for certain parameter ranges, with suction enhancing heat transfer and boundary layer stability, and stability analysis confirming that only the first solution branch is physically realizable.

Previous research by Mansur et al. (2014), Ho et al. (2021), and Ramzan et al. (2022) has explored various aspects of nanofluid flow and heat transfer. However, these studies primarily focus on either single nanoparticles or specific boundary conditions, and they lack a comprehensive analysis of the combined effects of suction and the enhanced thermal properties of hybrid nanofluids in porous environments. Additionally, the interactions between natural convection, forced convection, and the improved thermal conductivity of hybrid nanofluids have not been thoroughly examined. This knowledge gap restricts the potential optimization of hybrid nanofluids in engineering applications, such as cooling devices and porous heat exchangers, where high thermal efficiency is essential. To address this limitation, the present study investigates the impact of suction on the boundary layer development of hybrid nanofluid flow along a vertical surface embedded in a porous medium. By analyzing these interactions, this research aims to provide valuable insights into the enhancement of heat transfer technologies and the optimization of hybrid nanofluid-based systems for improved thermal performance in porous medium. Therefore, the purpose of this paper is to extend the work of Waini et al. (2020) by examining the effect of suction on hybrid nanofluid flow along a vertical surface embedded in a porous medium.

METHODOLOGY

The physical model considered in this paper is the two-dimensional mixed convection flow of a hybrid nanofluid past a vertical surface embedded in a porous medium with the effect of suction. The coordinate system is defined such that the x-axis is taken along the vertical surface and the y-axis normal to it. A schematic diagram of the physical model should be provided (Figure 1) to illustrate the boundary layer, suction velocity, and flow domain. The hybrid nanofluid is formed by dispersing copper (*Cu*) and alumina (Al_2O_3) nanoparticles in water as the base fluid.



(a) Assisting flow (b) Opposing flow

Figure 1: The physical model and coordinate system

The governing boundary-layer equations for continuity, momentum, and energy equations are given as follows (Merkin, (1980); Ahmad and Pop (2010)):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$U = U_\infty + \frac{gK(\rho\beta)_{hnf}}{\mu_{hnf}} (T - T_\infty), \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho c_P)_{hnf}} \frac{\partial^2 T}{\partial y^2}. \tag{3}$$

with the corresponding boundary conditions:

$$\begin{aligned} &v = V_w, T = T_w \;\; at \;\; y = 0 \\ &u \to U_\infty(x), T \to T_\infty \;\; as \;\; y \to \infty \end{aligned} \tag{4}$$

where u and v are velocity components in the x- and y-directions, respectively, T is the fluid temperature, U_∞ is the free stream velocity, T_w is the wall temperature, T_∞ is the ambient temperature, g is the gravitational acceleration, and K is the permeability parameter of the porous medium.

The thermophysical properties of nanofluids and hybrid nanofluids are presented in Table 1.

Table 1: Thermophysical properties of nanofluid and hybrid nanofluid

Properties	Nanofluid	Hybrid nanofluid
Density	$\rho_{nf} = (1-\varphi_1)\rho_f + \varphi_1\rho_{n1}$	$\rho_{hnf} = (1-\varphi_1)\left[(1-\varphi_1)\rho_f + \varphi_1\rho_{n1}\right] + \varphi_2\rho_{n2}$
Heat capacity	$(\rho C_p)_{nf} = (1-\varphi_1)(\rho C_p)_f + \varphi_1(\rho C_p)_{n1}$	$(\rho C_p)_{hnf} = (1-\varphi_2)[(1-\varphi_1)(\rho C_p)_f + \varphi_1(\rho C_p)_{n1}] + \varphi_2(\rho C_p)_{n2}$
Dynamic viscosity	$\mu_{nf} = \dfrac{\mu_f}{(1-\varphi_1)^{2.5}}$	$\mu_{hnf} = \dfrac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}$
Thermal conductivity	$k_{nf} = \dfrac{k_{n1} + 2k_f - 2\varphi_1(k_f - k_{n1})}{k_{n1} + 2k_f + \varphi_1(k_f - k_{n1})} \times (k_f)$	$\sigma_{hnf} = \dfrac{\sigma_{n2} + 2\sigma_{nf} - 2\varphi_2(\sigma_{nf} - \sigma_{n2})}{\sigma_{n2} + 2\sigma_{nf} + \varphi_2(\sigma_{nf} - \sigma_{n2})} \times (\sigma_{nf})$ $where$ $k_{nf} = \dfrac{k_{n1} + 2k_f - 2\varphi_1(k_f - k_{n1})}{k_{n1} + 2k_f + \varphi_1(k_{nf} - k_{n1})} \times (k_f)$
Electrical conductivity	$\sigma_{nf} = 1 + \dfrac{3(\frac{\sigma_{n1}}{\sigma_f} - 1)\varphi_1}{2 + \frac{\sigma_{n1}}{\sigma_f} - (\frac{\sigma_{n1}}{\sigma_f} - 1)\varphi_1} \times (\sigma_f)$	$\sigma_{hnf} = \dfrac{\sigma_{n2} + 2\sigma_{nf} - 2\varphi_2(\sigma_{nf} - \sigma_{n2})}{\sigma_{n2} + 2\sigma_{nf} + \varphi_2(\sigma_{nf} - \sigma_{n2})} \times (\sigma_{nf})$ $where$ $\sigma_{nf} = \dfrac{\sigma_{n1} + 2\sigma_f - 2\varphi_1(\sigma_f - \sigma_{n1})}{\sigma_{n1} + 2\sigma_f + \varphi_1(\sigma_{nf} - \sigma_{n1})} \times (\sigma_f)$
Thermal expansion efficient	$(\rho\beta)_{nf} = (1-\varphi_1)(\rho\beta)_f + \varphi_1(\rho\beta)_{n1}$	$(\rho\beta)_{hnf} = (1-\varphi_2)[(1-\varphi_1)(\rho\beta)_f + \varphi_1(\rho\beta)_{n1}] + \varphi_2(\rho\beta)_{n2}$

Here, φ_1 is the volume fraction of copper nanoparticles, and φ_2 is the volume fraction of alumina nanoparticles. Nanofluid is formed by disseminating copper, Cu in basic fluid such as water whereas hybrid nanofluid is formed by disseminating copper and alumina in basic fluid.

In order to solve the governing system of differential equations, we introduce the following dimensionless variables

$$\eta = \left(\frac{u_\infty}{2a_f x}\right)^{\frac{1}{2}} y\,, \qquad \psi = \left(2a_f U_\infty x\right)^{1/2} f\,(n)\,, \qquad T = (T_w - T_\infty)\theta(\eta) + T_\infty, \tag{5}$$

$$v_w = -\left(\frac{2U_\infty \alpha_f}{x}\right)^{\frac{1}{2}} S,$$

where ψ represents the stream function, η denotes the similarity variable, and T stands for the dimensionless temperature, and S is the suction parameter.

Substituting Eq.(5) into Eqs. (2)-(3), we obtain:

$$\frac{k_{hnf}/k_f}{(\rho c_P)_{hnf}/(\rho c_P)_f} f'''(\eta) + 2f(\eta)f''(\eta) = 0. \tag{6}$$

$$\frac{k_{hnf}/k_f}{(\rho c_P)_{hnf}/(\rho c_P)_f}\,\theta'' + 2\,f(\eta)\theta'(\eta) = 0. \tag{7}$$

The boundary conditions become:

$$f(0) = S, \qquad \theta(0) = 1, \qquad f'(0) = 1 + \frac{(\rho\beta)_{hnf}/(\rho\beta)_f}{\mu_{hnf}/\mu_f}\,\lambda, \tag{8}$$

$$f'(\eta) \to 1, \qquad \theta(\eta) \to 0 \; as\; \eta \to \infty$$

where $\lambda = Ra_x/Pe_x$ is the mixed convection parameter ($\lambda > 0$ assisting, $\lambda < 0$ opposing, $\lambda = 0$ forced convection).

The physical quantities of interest are the skin friction coefficient and local Nusselt number, defined as:

$$C_f = \frac{\tau_w}{\rho_f {U_\infty}^2}\,, \quad Nu_x = -\left(\frac{1}{\lambda}\frac{\frac{\mu_{hnf}}{\mu_f}}{(\rho\beta)_{hnf}/(\rho\beta)_f}\right) f''(0), \tag{9}$$

where $\tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is shear stress at the wall.

RESULTS AND DISCUSSION

The numerical results for the mixed convection flow of hybrid nanofluids along a porous vertical surface with suction are presented in this section. The governing equations (6) and (7) subject to the boundary conditions (8) were solved numerically using MATLAB's bvp4c solver, and the results are compared with previous studies to ensure the accuracy of the formulation.

To validate the present numerical procedure, we compared the skin friction coefficient for regular fluid with Waini et al. (2020) and Ahmad & Pop (2010) as presented in Table 2. The present findings are consistent with their reported trends, confirming the accuracy and reliability of the current model. In particular, the behaviour of velocity and temperature distributions under various convection parameters shows excellent agreement.

Table 2: Comparison of the results for skin friction coefficient for regular fluid ($\varphi_1 = \varphi_2 = 0$) with several values of λ.

λ	Ahmad & Pop (2010)		Waini et al. (2020)		Present result	
	First solution	Second solution	First solution	Second solution	First solution	Second solution
-1	0.46960		0.46960		0.46960	
-1.1	0.46105	0.00194	0.46105	0.00194	0.46105	0.00194
-1.15	0.44907	0.00866	0.44907	0.00866	0.44907	0.00866
-1.2	0.43015	0.02218	0.43015	0.02218	0.43015	0.02218
-1.25	0.40152	0.04539	0.40152	0.04539	0.40152	0.04539
-1.3	0.35664	0.08487	0.35664	0.08487	0.35664	0.08487
-1.35	0.25758	0.17856	0.25758	0.17856	0.25758	0.17856
-1.354	0.22428		0.22429	0.21143	0.22429	0.21143
-1.3541			0.21960	0.21611	0.21960	0.21611
-1.35411					0.21790	0.21800

Figures 2 and 3 present the velocity profiles, $f'(\eta)$ and temperature profiles, $\theta(\eta)$ for several values of the mixed convection parameter, λ with nanoparticle volume fractions $\varphi_1 = 0.1$ and $\varphi_2 = 0.1$ under suction condition ($S = 0.5$). The first solution shows an increase in temperature profiles and a decrease in velocity profiles with higher λ. Conversely, the second solution indicates a decrease in temperature profiles but an increase in velocity profiles. These findings highlight the sensitivity of the flow field and thermal boundary layer to variations in the mixed convection parameter. From the figures, it can be seen that dual solution exists for $\lambda < 0$.

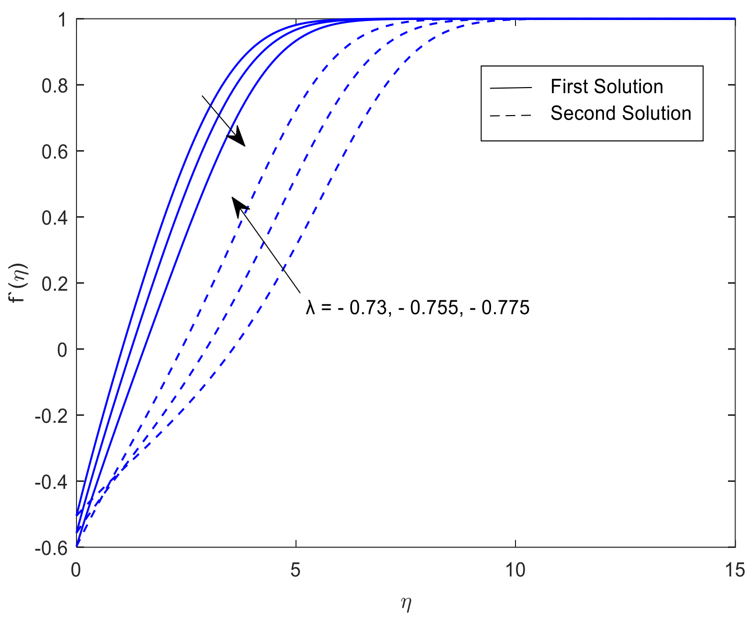


Figure 2: Velocity profiles $f'(\eta)$ for different values of the mixed convection parameter λ when $\varphi_1 = 0.1$ and $\varphi_2 = 0.1$ and suction parameter $S = 0.5$.

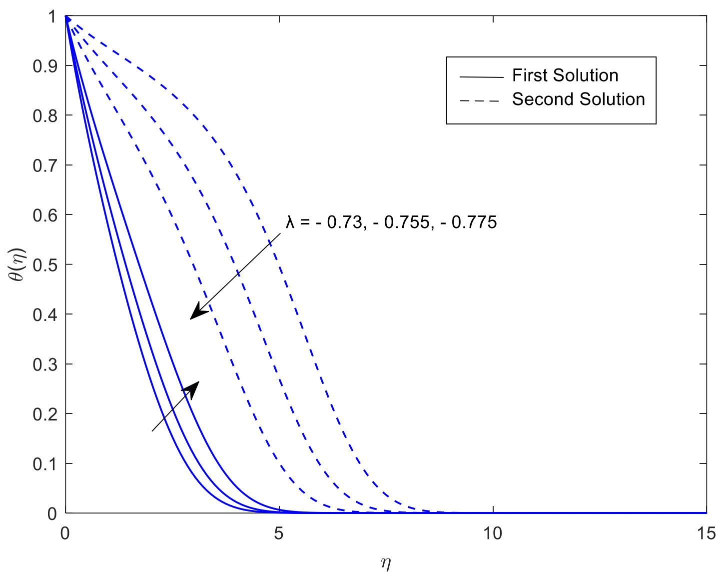


Figure 3: Temperature profiles $\theta(\eta)$ for different values of the mixed convection parameter λ when $\varphi_1 = 0.1$ and $\varphi_2 = 0.1$ and suction parameter $S = 0.5$.

Figures 4 and 5 illustrate the effect of the suction parameter on the skin friction coefficient, C_f and the reduced Nusselt number, $-\theta'(0)$. A unique solution exists for assisting flow, while dual solutions are found to exist for opposing flow where $\lambda_c < \lambda < 0$. As suction increases, the critical value range of the solution λ_c also increases, indicating a delay in boundary layer separation. Physically, higher suction stabilizes the boundary layer and strengthens the fluid adherence to the surface, thereby increasing C_f. Simultaneously, the reduced Nusselt number $-\theta'(0)$ increases with suction, suggesting improved heat transfer due to stronger temperature gradients near the wall.

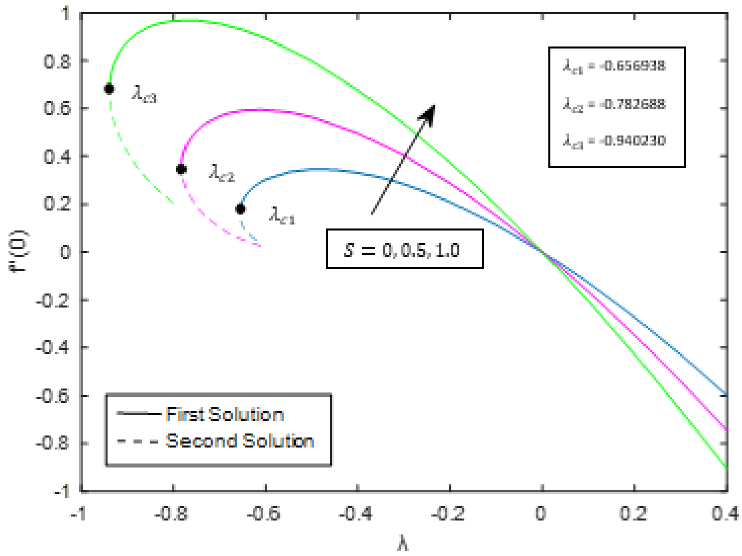


Figure 4: Variation of skin friction coefficient C_f with mixed convection parameter λ for several values of the suction parameter *S*.

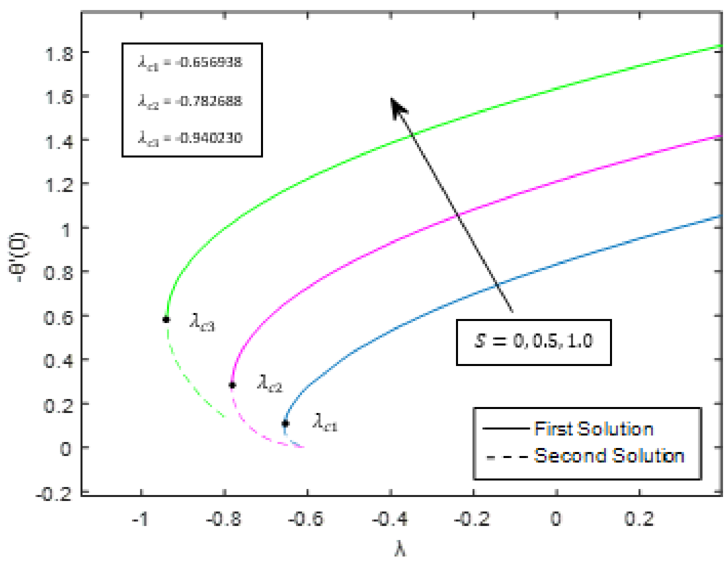


Figure 5: Variation of the reduced Nusselt number $-\theta'(0)$ with mixed convection parameter λ for several values of the suction parameter *S*.

CONCLUSION

This study demonstrates that suction enhances momentum and heat transfer in hybrid nanofluid flow along a porous vertical surface. Comparison results with Ahmad & Pop (2010) and Waini et al. (2020) validates the accuracy of the present numerical approach. Increased suction improves shear stress and heat transfer efficiency by stabilizing the boundary layer and enhancing fluid adherence to the surface. The mixed convection parameter *λ*, significantly influences flow stability and the thermal boundary layer thickness, with notable differences observed between assisting and opposing flow conditions. The presence of copper and alumina nanoparticles boosts thermal conductivity, although increased volume fractions lead to higher viscosity and boundary layer thickness. In this study, we can conclude that suction increases both the skin friction coefficient and the Nusselt number, confirming its stabilizing effect on the boundary layer. We observed that dual solutions exist for opposing flow. In general, optimizing suction and nanoparticle parameters is necessary to enhance thermal performance. These findings offer practical insights for developing efficient thermal management systems in porous media applications using hybrid nanofluids.

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